Journal of Statistical Physics, Vol. 49, Nos. 1/2, 1987

Collapse Transition and Cyclomatic Number Distribution of Directed Lattice Animals

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Received March 19, 1987; revision received May 21, 1987

We computed the specific heat of directed lattice animals using a Monte Carlo method for various animals sizes N, with N up to 100 on the square and N up to 125 on the simple cubic lattices. The specific heat as a function of the temperature for various animal sizes exhibits peaks which seem to approach a collapse transition temperature monotonically from below with increasing N. A least square fit together with finite-size scaling then gives both the transition temperature T_c and the specific heat exponent α for these two lattices. The cyclomatic number distributions for the number of animals with fixed animal size N are also calculated and these seem to obey a scaling law for large N.

KEY WORDS: Collapse transition; directed lattice animals.

1. INTRODUCTION

Connected clusters of occupied sites or lattice animals have been very well studied because of their relation to branched polymers and percolation theory.(1) The colapse transition of these large clusters has recently been studied by introducing attractive interaction between particles within the animals.(2–4) The attraction favors compact forms for the animals, while entropy works in the opposite direction to give ramified structures. This competition results in a collapse transition at a temperature T_c. The collapse transition in normal lattice animals had been studied by Derrida and Herrmann(2) in two dimensions using a transfer matrix method. They obtained both the transition temperature and the exponent for some two-

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dimensional lattices. Dickman and Schieve[3,4] studied the same problem using a Monte Carlo method for some two- and three-dimensional lattices. In two dimensions they obtained transition temperatures in close agreement with those of Derrida and Herrmann. They were not able, however, to determine the exponent.

A new version of the animal problem is the directed lattice animal problem[5-8] in which the elements (sites or bonds) are forbidden to grow against certain preferred directions in the lattice. The introduction of these preferred directions changes the universality class of the model. As far as we know, the collapse transition for these directed animals has not yet been studied. The purpose of this paper is to perform such studies for two- and three-dimensional lattices. We will show that the collapse transition problem in two dimensions is related to the directed percolation problem.

Thermal properties of lattice animals may be derived from the partition function

$$Z_N = \sum_B A'_N(B)\, e^{B/T} \tag{1}$$

in which $A'_N(B)$ is the number of animals of size N with B nearest neighbor bonds and T is the absolute temperature. We have expressed the energy in units such that $\varepsilon/k_B = 1$, where ε is the nearest neighbor attractive potential and k_B is Boltzmann's constant. Using Euler's relation $B = N + C - 1$, with C the number of cycles in the animal, we can rewrite Eq. (1) as

$$Z_N = e^{(N-1)/T} \sum_C^{M(N)} A_N(C)\, e^{C/T} \tag{2}$$

where $A_N(C)$ is the number of animals with exactly C cycles and $M(N)$ is the maximum number of cycles in animals of size N. For the suqare lattice, $M(N) = [2(N - \sqrt{N})] - N + 1$ and for the simple cubic lattice

$$\begin{aligned} M(N) = {} & [3(N - N^{2/3})] - N \\ & + \{\delta(N, L^3) + \delta(N, L^2(L+1)) + \delta(N, (L+1)^2 L)\} \end{aligned}$$

where the square brackets denote the largest integer of their arguments, δ is the Kronecker δ-function, and L is an integer. The Monte Carlo technique used by Dickman and Schieve[3] is modified and applied here to the directed problem. In this method, the ratios $r_N(C) = A_N(C)/A_N(C+1)$ for successive, nonzero values of $A_N(C)$ are estimated for fixed animal size N. In this way we have also calculated estimates for the cyclomatic number distribution $P_N(C)$ for animals of size N defined as

$$P_N(C) = A_N(C) \Big/ \sum_C^{M(N)} A_N(C) \tag{3}$$

Our numerical data for $P_N(C)$ suggest that $P_N(C)$ satisfies scaling law in both two and three dimensions. For fixed C, $A_N(C)$ behaves asymptotically for large N as(9–11)

$$A_N(C)=\sigma_C\lambda_0^N N^{C-\vartheta} \tag{4}$$

where λ_0 is a lattice-dependent growth parameter for lattice trees or zero cycle animals, ϑ is a universal exponent, and σ_C is an amplitude factor depending on C. Our estimates for $r_N(C)$ also allow us to investigate the amplitude factors σ_C. The result strongly suggests that for both the square and simple cubic lattices, the amplitude factors for different cycles are simply related.

2. MONTE CARLO METHOD FOR ESTIMATING THE NUMBER OF DIRECTED ANIMALS

In this section we review briefly the method of Dickman and Schieve(3) specialized to the directed animal problem. In this method one starts initially with a connected cluster of fixed size N connected to the site at the origin. One then generates different configurations of connected clusters of the same size N by removing randomly any of the $N-1$ sites, with the exception of the site at the origin, and attaching it at a randomly chosen unoccupied perimeter site, which in the directed case is an unoccupied successor to an occupied site. This unoccupied perimeter site is to be chosen at random from among the successor sites of a randomly selected occupied site. During this process one checks every time that the cluster does not break up into two or more disconnected clusters. Similar methods were used by Peters *et al.*(12) and Duarte(13) to generate lattice animals and lattice trees in the infinite-temperature limit.

In the case of directed animals, the connectivity constraint is easy to check. One has only to check that after the randomly chosen site is removed and attached to a randomly chosen perimeter site, all the successor sites of the removed site (in a fully directed d-dimensional hypercubic lattice there are most d such successor sites) are connected to at least one predecessor site. If the removed site has no successor site, then the removed site lies at the surface of the cluster and can always be removed and attached somewhere else without violating the connectivity constraint. After this new cluster configuration has passed the connectivity test, it is accepted with a probability p determined in the following way. Let $\mathbf{x}$ be the position of the site to be removed and $\mathbf{x}^*$ be the position of the site to be occupied. We denote the new configuration Γ' to be the one in which $\mathbf{x}^*$ is occupied and $\mathbf{x}$ is empty and the old configuration Γ as the reversed situation. One determines in the new configuration Γ' the number of predecessors n'_p of **x** and the number of bonds n'_b (i.e., the sum of the number of predecessor an successor sites) of **x*** and similarly in the old configuration Γ the number of predecessors n_p of **x*** and the number of bonds n_b of **x**. If n'_b is greater than or equal to n_b, the new configuration is accepted with probability $p = n'_p/n_p$. Otherwise it is accepted with probability $p = \exp[(n'_b - n_b)/T]$, where T is the temperature. It is easy to show that this choice of p gives the transition probability $R(\Gamma \to \Gamma')$ satisfying detailed balance.

Consider the transition probability from the old configuration Γ to the new configuration Γ'. In the old configuration Γ, the probability of choosing **x** from among $N-1$ occupied sites is $(N-1)^{-1}$ and the probability for choosing another occupied site, say **y**, from which the perimeter site **x*** is to be chosen, from among N occupied sites is N^{-1}. Since in the old configuration Γ the number of predecessor sites of **x*** is n_p, the probability for choosing **x*** from **y** is just $2n_p/q$, where q is the coordination number of the lattice, the factor two entering because we are considering directed animals. With our above choice of p, the transition probability $R(\Gamma \to \Gamma')$ is given by

$$R(\Gamma \to \Gamma') = \begin{cases} 2n_p(n'_p/n_p)/qN(N-1), & n'_b \geqslant n_b \\ 2n_p \exp[(n'_b - n_b)/T]/qN(N-1), & n'_b < n_b \end{cases} \tag{5a}$$

Similarly, the reversed probability $R(\Gamma' \to \Gamma)$ is given by

$$R(\Gamma' \to \Gamma) = \begin{cases} 2n'_p(n_p/n'_p)/qN(N-1), & n_b \geqslant n'_b \\ 2n'_p \exp[(n_b - n'_b)/T]/qN(N-1), & n_b < n'_b \end{cases} \tag{5b}$$

Therefore, for $n'_b \geqslant n_b$,

$$R(\Gamma \to \Gamma') = 2n'_p/qN(N-1)$$
$$R(\Gamma' \to \Gamma) = 2n'_p \exp[(n_b - n'_b)/T]/qN(N-1)$$

This implies that for $n'_b > n_b$,

$$R(\Gamma \to \Gamma') \exp(-n'_b/T) = R(\Gamma' \to \Gamma) \exp(-n_b/T)$$

Exactly the same result is obtained for $n'_b < n_b$. This shows that $R(\Gamma \to \Gamma')$ satisfies detailed balance. An unbiased estimate of the ratio $A_N(C)/A_N(C')$ is then given by $\exp[(C'-C)/T] Y_C/Y_C$, where Y_C is the number of realizations of C cycle animals. For large animals, e.g., $N = 64$ or $N = 100$ in the square lattice, animals with small C are generated only with sufficiently large T and those with very large C are generated only with sufficiently small T. Therefore one has to use several different temperatures (four or five) in order to generate animals with cycle numbers ranging all the way from $C=0$ to $C=M(N)$, the maximum allowed number of cycles for animals of size N. For each temperature we make over 100 million trials at generating new configurations starting from an initial connected cluster. An accepted configuration is called an event. The number of events may be of order 10 million for T not too low. For lower T, the number of events can be considerably smaller. At the start of each run 10 or 20 million trials are allowed for relaxation. For $N \leqslant 16$ on the square and $N \leqslant 12$ on the simple cubic lattices, we have checked that our Monte Carlo data agree excellently with exact results.(14)

3. LATTICE ANIMAL SPECIFIC HEATS

The specific heat per particle in an N-particle animal is

$$C_v = (\langle B^2 \rangle - \langle B \rangle^2)/NT^2 \sim (T-T_c)^{-\alpha} \tag{6}$$

where

$$\langle B^n \rangle = Z_N^{-1} \sum_B B^n A'_N(B)\, e^{B/T}$$

and $A'_N(B)$ is the number of animals of size N with B bonds. If $A_N(C) \neq 0$, $\forall C \leqslant M(N)$, then defining $r_N(C) \equiv A_N(C)/A_N(C+1)$, we can write Z_N as

$$\begin{aligned} Z_N = e^{(N-1)/T} A_N(0) \Bigg\{ & 1 + e^{1/T}/r_N(0) + e^{2/T}/r_N(0)\, r_N(1) + \cdots \\ & + \prod_{C=0}^{M(N)-1} [e^{1/T}/r_N(C)] \Bigg\} \end{aligned} \tag{7}$$

and similarly for $\langle B^n \rangle$,

$$\begin{aligned} \langle B^n \rangle = & \left\{ (N-1)^n + \sum_{C=0}^{M(N)-1} (N+C)^n \prod_{j=0}^{C} [e^{1/T}/r_N(j)] \right\} \\ & \times \left\{ 1 + \sum_{C=0}^{M(N)-1} \prod_{j=0}^{C} [e^{1/T}/r_N(j)] \right\}^{-1} \end{aligned} \tag{8}$$

For the square lattice we find that $A_N(C) \neq 0$, $\forall C \leqslant M(N)$. We have calculated $r_N(C)$ using the Monte Carlo method of the last section, for $N = 25, 36, 64, 100$. The specific heat of directed square lattice animals obtained using (6) and (8) is plotted as a function of T in Fig. 1 for various sizes N. The result for $N = 16$ is exact. From Fig. 1 we see that besides the

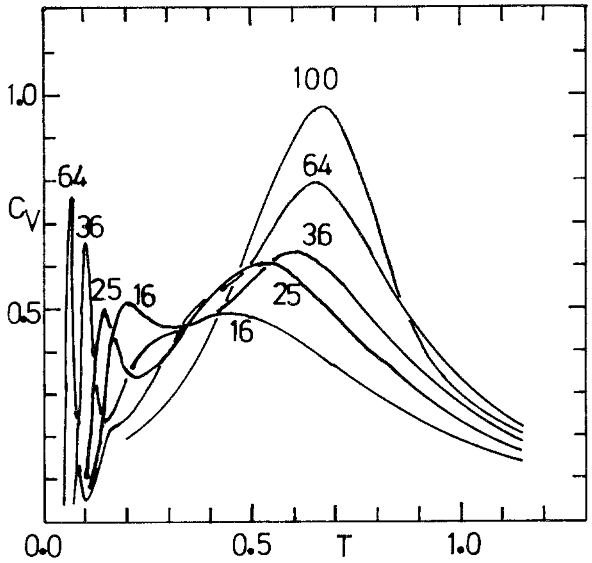

Fig. 1. Specific heat C_v against temperature T for various animal sizes N on the square lattice.

subsidiary peak at low temperatures, the specific heat also exhibits a strong, fairly broad peak at a higher temperature. We identify this as the collapse point $T_m(N)$ for directed animals of finite size N. As the size N of the animals increases, the temperature $T_m(N)$ at which the specific heat attains its maximum shifts toward higher temperatures and the peak grows higher and sharper. For the square lattice, with $N = 100$, $T_m(N) = 0.68$. From finite size scaling(15) we know that $T_m(N)$ scales as

$$T_m(N) = T_c - aN^{-\phi} \tag{9}$$

with a crossover exponent ϕ. We have used our data $T_m(16) = 0.45$, $T_m(25) = 0.52$, $T_m(36) = 0.60$, $T_m(64) = 0.65$, and $T_m(100) = 0.68$ in a least square fit to determine the three parameters T_c, ϕ, and a in (9). We find $T_c = 0.77$ and $\phi = 0.702$. In Fig. 2 we plot $T_m(N)$ versus $N^{-\phi}$ for directed square lattice animals, using the crossover exponent value $\phi = 0.702$. We see that the data fall fairly well on a straight line, confirming finite-size scaling theory. Our values for T_c are somewhat higher than the normal, undirected lattice animal value $T_c = 0.535$, but the crossover exponent is not essentially different from the corresponding, normal animal value of $\phi = 0.657$.(2) The specific heat exponent α can be related to the crossover exponent ϕ as follows.(2) If the thermal correlation length ξ_T of directed lattice animals near the collapse point goes as

$$\xi_T \sim |T - T_c|^{-\nu_2} \tag{10}$$

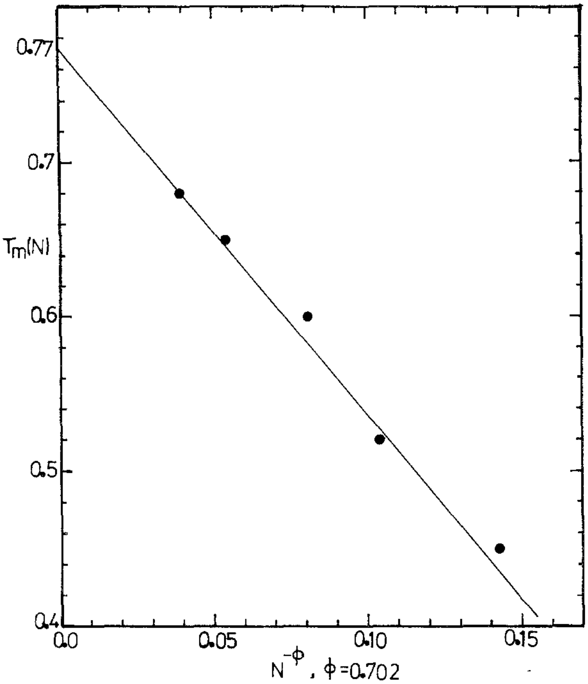

Fig. 2. Variation of $T_m(N)$ with size for square lattice animals. The straight line is obtained by a least square fit.

with an exponent ν_2, then the free energy per particle f is given by

$$f \sim \xi_T^{-\bar{d}} \sim |T - T_c|^{\bar{d}\nu_2} \tag{11}$$

where $\bar{d}$ is the fractal dimension of the directed lattice animals at the collapse point. From (11) it follows that the free energy f also has the exponent $2 - \alpha$. Therefore we have

$$\bar{d}\nu_2 = 2 - \alpha \tag{12}$$

If we assume that at the collapse point the radius of gyration exponent of directed lattice animals is ν_1 (i.e., we do not have to distinguish between parallel and transverse exponents $\nu_\parallel$ and $\nu_\perp$ at the collapse point, in contrast to the case for infinite-temperature directed lattice animals), we have $\bar{d} = 1/\nu_1$ and

$$\alpha = 2 - \nu_2/\nu_1 = 2 - 1/\phi \tag{13}$$

with $\phi = \nu_1/\nu_2$, the same crossover exponent in (9).

For the simple cubic lattice with $N = L^3$, where L is an integer, we find that $A_N(M(N) - 1) = 0$ for all the N values we have investigated. Since then $r_N(M(N) - 2)$ is not defined, we have to modify (8) to

$$\begin{aligned}\langle B^n \rangle = \Bigg\{ &(N-1)^n + \sum_{C=0}^{M(N)-3} (N+C)^n \prod_{j=0}^{C} [e^{1/T}/r_N(j)] \\ &+ [N + M(N) - 1]^n \left[\prod_{C=0}^{M(N)-3} e^{1/T}/r_N(C) \right] e^{2/T}/r'_N \Bigg\} \\ &\times \Bigg\{ 1 + \sum_{C=0}^{M(N)-3} \prod_{j=0}^{C} [e^{1/T}/r_N(j)] \\ &+ \left[\prod_{C=0}^{M(N)-3} e^{1/T}/r_N(C) \right] e^{2/T}/r'_N \Bigg\}^{-1} \end{aligned} \tag{8'}$$

where

$$r'_N \equiv A_N(M(N) - 2)/A_N(M(N)) \tag{14}$$

The specific heat of a simple cubic directed lattice animal obtained using (6) and (8′) [except for $N = 96$, where we still use (8)] is plotted as a function of T in Fig. 3 for $N = 27$, 64, 96, and 125. We find the main peak of the specific heat at $T_m(27) = 0.45$, $T_m(64) = 0.83$, $T_m(96) = 0.89$, and

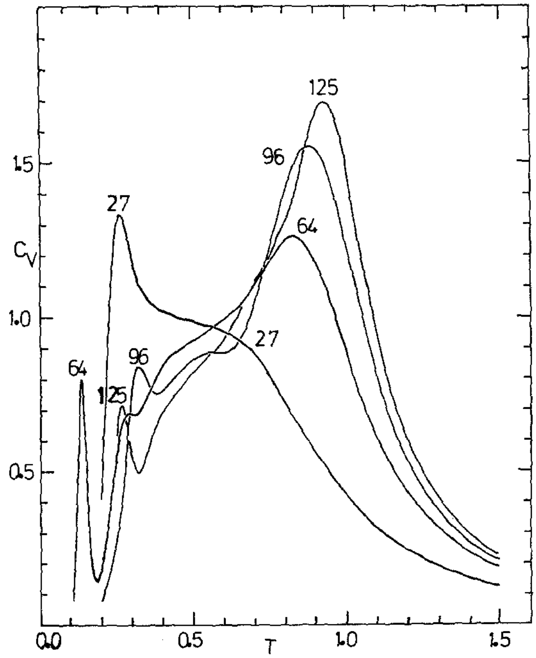

Fig. 3. Same as Fig. 1, but for the simple cubic lattice.

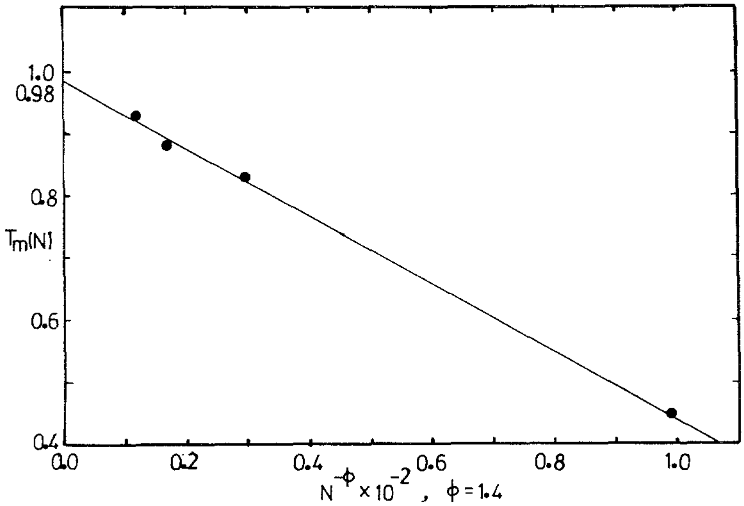

Fig. 4. Same as Fig. 2, but for the simple cubic lattice.

$T_m(125) = 0.93$. For $N = 27$ we have taken the main peak to be at the inflection point. We observe that $T_m(N)$ increases monotinically with N, i.e., the collapse point T_0 is approached from below. The peaks also become sharper with increasing N. Using (9), we have estimated T_c and ϕ with a least square fit: $T_c = 0.984$, $\phi = 1.4$. Using (13), this corresponds to a specific heat exponent $\alpha = 1.286$. In Fig. 4 we plot $T_m(N)$ as a function of $N^{-\phi}$, with $\phi = 1.4$. The data fall reasonably well on a straight line, again confirming finite-size scaling theory.

4. CYCLOMATIC NUMBER DISTRIBUTION

The cyclomatic number distribution defined in (3) can be written in terms of the $r_N(C)$ when $A_N(C) \neq 0$, $\forall C \leqslant M(N)$, as

$$P_N(C) = \left\{ 1 + \sum_{C'=0}^{C-1} \prod_{j=C'}^{C-1} r_N(j) + \sum_{C'=C}^{M(N)} \prod_{j=c}^{C'-1} r_N(j)^{-1} \right\}^{-1}, \qquad 0 < C < M(N) \tag{15}$$

In the case of the simple cubic lattice, $A_N(M(N) - 1) = 0$ when $N = L^3$, where L is an integer. Then (15) is modified to

$$P_N(C) = \left\{ 1 + \sum_{C'=0}^{C-1} \prod_{j=C'}^{C-1} r_N(j) + \sum_{C'=C}^{M(N)-2} \prod_{j=C}^{C'-1} r_N(j)^{-1} + \left[\prod_{j=C}^{M(N)-3} r_N(j)^{-1} \right] r_N'^{-1} \right\}^{-1}, \qquad 0 < C < M(N) \tag{15'}$$

where r'_N is defined in (14). We have calculated $P_N(C)$ for directed lattice animals on both the square and the simple cubic lattices using (15) or (15′). The result is most conveniently presented in terms of the quantity

$$Q_N(X) \equiv \log P_N(C)/\log P_N(M(N))$$

as a function of the variable $X \equiv C/M(N)$. Figures 5 and 6 show the quantity $Q_N(X)$ as a function of X for the square and simple cubic lattices, respectively. From these figures, we notice that there is evidence of approximate scaling as observed by Dickman and Schieve[14] for undirected lattice animals. From (3), since $A_N(C)$ can be written as

$$A_N(C) = A_N(0) \prod_{j=0}^{C-1} r_N(j)^{-1} \tag{16}$$

we see that the errors in $P_N(C)$ increase with C: Let Δr_j be the statistical error in $r_N(j)$; then

$$\prod_{j=0}^{C-1} [r_N(j) + \Delta r_j]^{-1} \approx \left[1 - \sum_{j=0}^{C-1} \Delta r_j/r_N(j)\right] \prod_{j=0}^{C-1} r_N(j)^{-1} \tag{17}$$

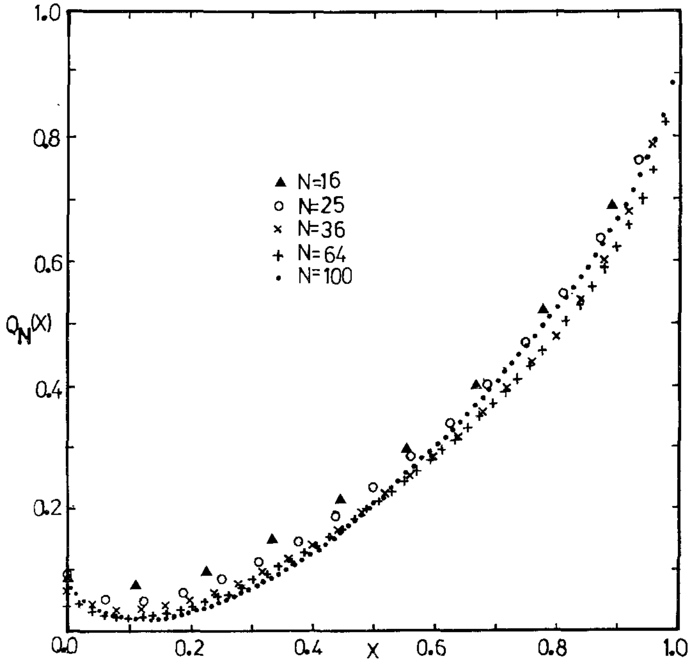

Fig. 5. Quantity $Q_N(X)$ versus X for directed lattice animals on the square lattice.

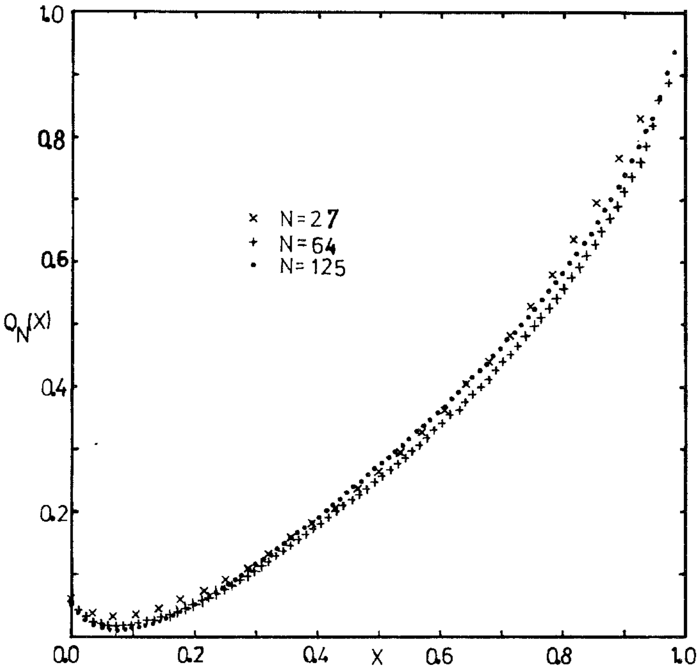

Fig. 6. Same as Fig. 5, but for the simple cubic lattice.

We have in our calculation $\Delta r_j/r_N(j) \approx 0.01$. At $X = 1$, $Q_N(X)$ is normalized to 1 for all N. For small X we see from Figs. 5 and 6 that there is very good scaling behavior. For X between 0.5 and 0.9 the scaling behavior is not so good. But then the error bars in this region are also larger, so that the data are not inconsistent even with the conjecture of an exact scaling behavior for all X at large N.

For fixed cycle number C it is known(9-11) that $A_N(C)$ has the following asymptotic behavior for large N:

$$A_N(C) = \sigma_C \lambda_0^N N^{C-\vartheta}, \qquad N \gg 1 \tag{18}$$

where σ_C is a cycle-dependent amplitude factor, λ_0 is the growth parameter for lattice trees, i.e., $C = 0$ lattice animals, and ϑ is a universal exponent. From (18) we see that the ratio of successive amplitude factors σ_C/σ_{C+1} can be obtained from

$$\sigma_C/\sigma_{C+1} = NA_N(C)/A_N(C+1) = Nr_N(C) \tag{19}$$

In Figs. 7 and 8 we plot the successive ratios $NA_N(C)/A_N(C+1)$ for the square and simple cubic lattices, respectively, as a function of C. These two

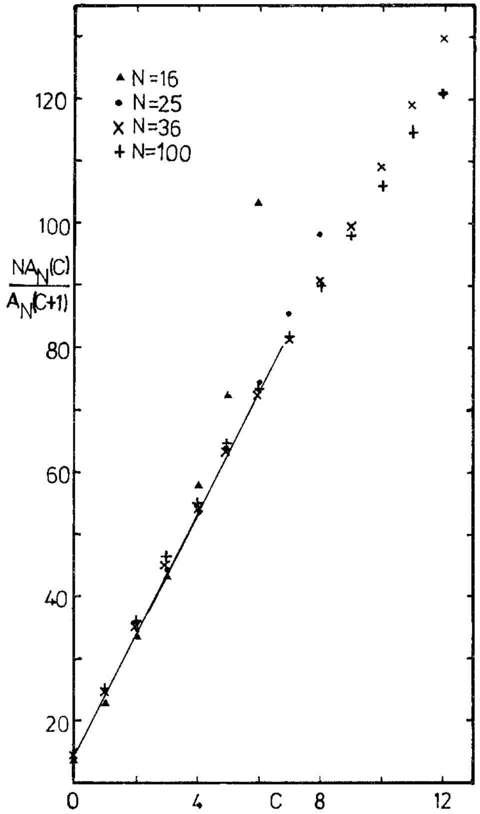

Fig. 7. Ratio $NA_N(C)/A_N(C+1)$ of the number of directed animals with fixed size N but successive cycle numbers C and $C+1$ as a function of C for the square lattice.

figures strongly suggest the following relation between ratios of the successive amplitude factors σ_C/σ_{C+1}:

$$-\sigma_C/\sigma_{C+1}+\sigma_{C+1}/\sigma_{C+2}=K \tag{20}$$

where K is a constant. We find $K\approx 10$ for the square and $K\approx 9$ for the simple cubic lattices, respectively. In the following we will give a justification for relation (20). As a consequence of (20), we have

$$\sigma_C=\sigma_0(\sigma_1/\sigma_0)^C\Big/\prod_{j=1}^{C-1}(1+jK\sigma_1/\sigma_0) \tag{21}$$

with $(\sigma_1/\sigma_0)\approx 1/14$ for the square and $(\sigma_1/\sigma_0)\approx 1/11$ for the simple cubic lattices, taken from Figs. 7 and 8. If we sum (18) over all cycles C, we

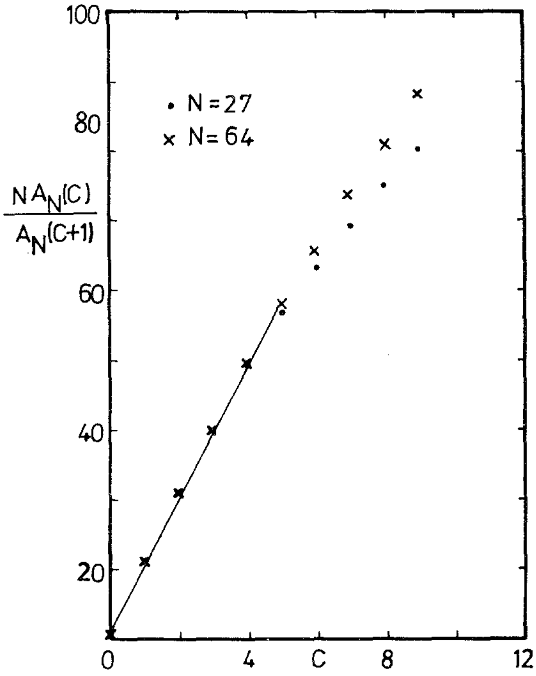

Fig. 8. Same as Fig. 7, but for the simple cubic lattice.

obtain the total number of animals of size N, A_N, which has the asymptotic behavior at large N:

$$A_N = \sigma \lambda^N N^{-\vartheta} \tag{22}$$

where λ is the growth parameter for the total number of directed lattice animals and ϑ is the same universal exponent as in (18). We have therefore the equation

$$(\sigma_0/\sigma)(1 + N\sigma_1/\sigma_0 + N^2\sigma_2/\sigma_0 + \cdots + N^{M(N)}\sigma_{M(N)}/\sigma_0) = (\lambda/\lambda_0)^N \tag{23}$$

In the limit $N \to \infty$, the left-hand side of (23) must tend toward an exponential function $\exp(aN)$, with $a = \log(\lambda/\lambda_0)$ and $\sigma_0 = \sigma$. If we substitute (21) into (23), we have

$$(\lambda/\lambda_0)^N = \sum_{C=0}^{M(N)} (N\sigma_1/\sigma_0)^C/C! \prod_{n=1}^{C} n[1 + (n-1)K\sigma_1/\sigma_0]^{-1} \tag{24}$$

Now since $K\sigma_1/\sigma_0 \approx 1$, we have for n large,

$$n[1 + (n-1)K\sigma_1/\sigma_0]^{-1} \approx (K\sigma_1/\sigma_0)^{-1}$$

Therefore (2) becomes

$$(\lambda/\lambda_0)^N \approx \sum_{C=0}^{M(N)} (N/K)^C/C! \approx \exp(N/K) \tag{25}$$

with $K^{-1} = \log(\lambda/\lambda_0)$. For the square lattice we have the exact result $\lambda = 3$,(4,5) and the very accurate result $\lambda_0 = 2.71261$ from transfer matrix calculation.(7) We have therefore in this case $\log(\lambda/\lambda_0) = 0.1$, in very good agreement with our square lattice result $K \approx 10$. Actually, (20) seems to be correct also for normal undirected lattice animals.(16) A particular solution of (20) is the Poisson distribution $\sigma_C = \sigma_0/(K^C C!)$. Substituting this into (18) and summing over all C, up to $C = M(N)$, we obtain, in the limit $N \to \infty$, again $K^{-1} = \log(\lambda/\lambda_0)$. Equation (21) reduces to the above Poisson distribution if $\sigma_0 = K\sigma_1$. But in general (20) alone does not give such a relation between σ_1 and σ_0.

5. RELATION TO DIRECTED PERCOLATION IN TWO DIMENSIONS

We have been informed by Dhar(17) of a relation between the collapse transition of directed lattice animals and directed bond percolation in two dimensions. Consider directed bond percolation on a square lattice. It is easy to see that if p is the bond occupation probability, then the probability of generating a cluster is

$$q^t p^m (1 - q^2)^n = q^t p^m (2pq - p^2)^n$$

where $q \equiv 1 - p$, t is the number of perimeter bonds, m is the number of sites with one predecessor, and n is the number of sites with two predecessors. But $m = N - C$, $n = C$, and $t = N - C + 1$, where N is the number of sites and C is the number of loops in the cluster. Therefore the probability of generating a cluster is given by

$$q^{N-C+1} p^{N-C} (2pq - p^2)^C \sim [(2pq - p^2)/pq]^C$$

Comparing with (2), we have

$$e^{1/T} = (2pq - p^2)/pq = (1 + q)/q$$

The critical points of the square lattice directed percolation problem are $q_c = 1$, 0.3553, and 0.(8) The first corresponds to the empty lattice, the second corresponds to the percolation threshold, and the third corresponds to the compact lattice. For $q_c = 0.3553$ we have $T_c = 0.747$, in reasonable agreement with our square lattice result. At $q_c = 0$, $T_0 = 0$. This corresponds to the appearance of the subsidiary peak shown in Fig. 1. This peak seems to shift toward $T_0 = 0$ as N increases. It presumably describes the roughening transition, as discussed by Dickman and Schieve for normal, undirected lattice animals.(3) This conjecture is supported by the following arguments. First, the roughening transition in two-dimensional systems does occur at $T = 0$, just as observed here. Second, the subsidiary peak is present only for $N = L^2$, with L an integer in the square lattice. This can be understood by observing that when $N \neq L^2$ on the square lattice, the maximally bonded state is degenerate and has steps on the surface, so that in effect the surface is already somewhat rough even at $T = 0$. For the simple cubic lattice the subsidiary peak is present either when $N = L^3$ or when N is such that the maximally bonded state is in the form of a parallelepiped (a matchbox shape), e.g., in our case when $N = 96$. But the subsidiary peak for the simple cubic lattice fluctuates around a finite temperature $T \approx 0.2$ as the size N increases. The roughening transition, if such is the case, now occurs at a finite temperature for the simple cubic lattice.

6. CONCLUSION

We have calculated the collapse transition temperature and specific heat exponents for the directed lattice animals on the square and simple cubic lattices. In addition, the cyclomatic number distributions were also calculated for these two lattices. They seem to obey a scaling law for large N. We have also shown the relation of the collapse transition in directed lattice animals to directed percolation in two dimensions. This yields a collapse transition temperature in close agreement with our Monte Carlo result and presumably also a roughening transition at $T = 0$, which is also in agreement with the behavior of the subsidiary peak in our Monte Carlo specific heat data for the square lattice. We have also shown that at fixed cycle number C, the number of animals of size N is given by the asymptotic formula $A_N(C) = \sigma_C \lambda_0^N N^{C-\vartheta}$, where the amplitude factors σ_C for different C are simply related to one another.

ACKNOWLEDGMENT

We thank D. Stauffer for discussion and D. Dhar for useful correspondence. We are also grateful to the Sonderforschungsbereich 125 for financial support.

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